La Quinta Motor Inns is a mid-sized hotel chain headquartered in San Antonio, Texas. They are looking to expand to more locations, and know that selecting good sites is crucial to a hotel chain's success. Of the four major marketing considerations (price, product, promotion, and location), location has been shown to be one of the most important for multisite firms.

Hotel chain owners who can pick good sites quickly have a distinct competitive advantage, since they are competing against other chains for the same sites. La Quinta used data on 57 existing inn locations to build a linear regression model to predict "Profitability", computed as the operating margin, or earnings before interest and taxes divided by total revenue. They tried many independent variables, such as "Number of hotel rooms in the vicinity" and "Age of the Inn". All independent variables were normalized to have mean zero and standard deviation 1.

The final regression model is given by:

Profitability = 39.05 - 5.41\*(State Population per Inn) + 5.86\*(Price of the Inn) - 3.09\*(Square Root of the Median Income of the Area) + 1.75\*(College Students in the Area)

The R-squared of the model is 0.51.

In this problem, we'll use this regression model together with integer optimization to select the most profitable sites for La Quinta.

Problem 1- Selecting the Most Profitable Hotels

According to the regression equation given above, which variables positively affect Profitability?

Answer: Price of the Inn, College Students in the Area

Using this regression equation, La Quinta created a spreadsheet model to predict profitability, and routinely uses it to screen potential real estate acquisitions. Suppose that La Quinta is looking to expand their locations in California, and has collected data for 16 different potential sites. This data is given in the spreadsheet [SelectingHotels.xlsx](https://prod-edxapp.edx-cdn.org/assets/courseware/v1/7bc694fbe16762a5e42aafab6000e5fb/asset-v1:MITx+15.071x+2T2017+type@asset+block/SelectingHotels.xlsx) for Microsoft Excel. For each hotel, it lists the location of the hotel, the price, and the value for each of the independent variables used in the regression equation (normalized to have mean zero and standard deviation one).

Using the regression equation, what is the predicted profitability of hotel 1?

Answer: 39.05 - 5.41\*(-1) + 5.86\*(-0.3) - 3.09\*(-0.81) + 1.75\*(-0.54) = 44.26

In your spreadsheet, compute the predicted profitability for all hotels.

Which hotel has the highest predicted profitability?

Answer: Hotel 2

Which hotel has the lowest predicted profitability?  
Answer: Hotel 8

La Quinta has a budget of $10,000,000 to spend on hotels. Suppose we just used a "greedy" approach where we selected the most profitable hotels until we ran out of budget. So we would start by buying the hotel we predict to be the most profitable, and then if we had enough budget left, we would buy the hotel we predict to be the second most profitable, etc.

How many hotels would we purchase with this approach?

Answer: Hotel 2 has the highest predicted profit but also costs $10,000,000, so we would buy just this one hotel.

What would our total predicted profitability be?

Answer: 53.3791923

Problem 2- An Optimization Approach

Now, build an optimization model in your spreadsheet to select hotels. The decision variables are whether or not a hotel is selected (binary variables). The objective is to maximize the total predicted profitability. We have two constraints: the decision variables should be binary, and the total cost should not exceed the budget of $10,000,000. Formulate and solve this model in Excel.

What is the objective value of the solution?

Answer: $269.92

How many hotels are selected in the solution?

Answer: 7

How many hotels located in South Lake Tahoe are selected in the solution?

Answer: 6

La Quinta thinks that buying too many hotels in one city is probably not a good idea, and would prefer to diversify in other cities, even though it will decrease the sum of the predicted profitability. Add a constraint to limit the number of hotels selected in South Lake Tahoe to 2.

What is the objective value of the solution now?

Answer: $205.70

How many hotels (in total) are selected in the solution now?

Answer: 6

In which cities do we buy at least one hotel?

Answer: Eureka, Fresno, Los Angeles, South Lake Tahoe

In this problem, we compared the greedy approach with an optimization approach, and saw that the optimization approach was much better. This is true in many situations, but not always. In which of the following situations would the greedy approach perform as well as the optimization approach?

i) Instead of maximizing the sum of the profitability of the hotels we select, we wanted to maximize the average profitability of the hotels we select.

ii) Instead of having a budget constraint, we had a constraint on the number of different hotels we can select (for example, we want to maximize profitability given that we can only select 2 hotels).

iii) Instead of having a budget of $10,000,000, we had a budget of $20,000,000.

Answer: i) Would be the same in both cases. If we want to maximize the average profitability then this amounts to choosing the single most profitable hotel. ii) Would be the same in both cases because both approaches would just pick the k most profitable hotels, where k is the number of hotels picked in each case. iii) Would not be the same in both cases; following the computations as we did in the $10,000,000 case above we would find that the greedy approach is much less profitable.